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The Influence of Population Growth on Per-Worker Income in Developed Economies

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growth (see Sauvy, 1969; Kuznets, 1967; Easterlin, 1967). Growth in knowledge caused by the larger number of people is the main factor that has been adduced to explain the discrepancy between the classical model and the observed reality (Kuznets, 1960).

The aim of this paper is to create a framework within which the contradiction may be reconciled quantitatively. Knowledge and related factors are incorporated into the standard analysis so that the opposing strengths of growth in knowledge and of diminishing returns may be weighed against each other. Crude estimates are inserted into iterative simulations to learn the effect on per-worker income of various patterns of population growth under various conditions. Within the range of assumed conditions, the effect of a population increase generally goes from negative to positive within 50 to 100 years. It must be emphasized, however, that the various models simulated are intended only to be illustrative and suggestive, and do not purport to represent either the U.S., or any other single country, or the developed world as a whole. Also to avoid disappointment, please be warned that the central finding results from adding a single element -- the effect of population size upon productivity -- to a simple classical model, within what seem to the writer to be reasonable ranges of the basic parameters. The outcome of adding this factor may seem obvious, but I believe that the elaboration of the models is necessary to make the central finding seem persuasive as a description of economic and population growth in the more-developed world.

The paper may be viewed as an attempt to formalize and quantify* Kuznets' masterful paper (1960). In the formal approach used here many of the interesting influences discussed by Kuznets are necessarily left out; the reader is therefore referred to that paper for greater richness and a more complex view of the matter. It should be noted, however, that this paper is not an empirical study. It is, rather, a theoretical exercise in which the simulation technique is used instead of analytic methods. This has the disadvantage of less generality than analytic methods, because the results hold only for the specific sets of parameters the models are run on, and apply only by analogy to other sets of parameters within the ranges of the simulated sets. That is, unlike analytic methods, no results are proven to hold for all cases consistent with the basic assumptions. On the other hand, the simulation method has the advantage of allowing one to theorize about a much richer and more realistic model than analytical methods allow, and with more specificity.

The context of the paper is near-full employment; when there is substantial unemployment, population growth is relatively more favorable because of the positive effect on demand, including the public sector. The time horizon is sufficiently short so that possible major changes in the natural-resource situation may be disregarded -- perhaps 50

*"...we have no tested, or even approximate, empirical coefficient with which to weight the various positive and negative aspects of population growth." (Kuznets, 1960, p. 339)

or 150 years. Though the terms of reference are to the United States, it would be most appropriate to conduct this analysis for the developed world as a whole, because of the scientific and technological interdependence among the MDC's.

The aim here is not to determine whether it is worthwhile for a society to have more or fewer children. Such a judgment would require various difficult assumptions such as the following: (a) A rate at which future consumption utility is to be discounted in the present must be chosen if a welfare decision is to be made. A very high discount rate implies that having no children is best, because the children will become producers only long after they are consumers; and a zero discount rate makes any meaningful calculation impossible. (b) A welfare decision requires that one decide which people are to be considered the members of the community whose welfare is to be maximized. For example, should one maximize the welfare of only the people alive today, or should one also include future peoples' welfare in the objective function? (c) One would need to make difficult decisions about what is to be considered consumption. For example, which parts of a child's education in the home,

street, and school are to be considered investment, and which are to be considered consumption by the child, his parents and society?

The purposes of this paper rather, are these: (1) to understand in the history of industrial nations the influences of population growth on income through changes in capital and knowledge; (2) to consider what the future course of output per worker might be with higher or lower birth rates.

The dependent variable is "output (or income) per worker,"* and not consumption per capita (or per consumer-equivalent). In the long run the two measures are much the same. In the short run an increase in population through an increase in fertility necessarily implies a drop in consumption per capita even if output per worker remains the same, because the total number of workers remains the same while the number of people increases. In the household, income is then spread among more people. And when population grows faster there is greater public consumption of education and other child-raising services,** which implies larger taxes and less resources available for private consumption and saving. But in the long run, measures of consumption per capita and output per worker will give much the same result, and the focus here is

*It is not assumed here that per-capita income is the appropriate measure of welfare; elsewhere I argue that it is not (Simon, 1970). But per-capita income is one of the arguments in almost everyone's welfare function.

**The investment aspect of education will be treated later.

on the long run. Furthermore, lower per-person consumption need not mean lower total utility. In fact, depending on one's social welfare function, the same total consumption spread among more people might be seen as yielding higher total utility.

The first section after this introduction gives the demographic structures to be analysed. The second section sets forth a simple classical analysis to estimate the partial effect of a higher birth rate on income through the capital stock, savings, and labor force. The third section describes a way to estimate the partial effect of a higher birth rate through the variables not included in the classical model, i.e., knowledge, scale effects, and natural resources. In the fourth section the two partial approaches are brought into numerical relation with each other to weigh the net effect on per-worker income in a large rich industrialized country of more or less children being born.

The symbols used are as follows:

A_t = level of the economy's productive efficiency as of year t .

e_L = elasticity of labor force with respect to children born

e_s = elasticity of the saving ratio, s , with respect to number of children

$F_{i,t}$ = number of females of age i as of year t

H_t = the stream of increments to productivity caused by an increment to knowledge in year t created by a given increment of people

K_t = stock of capital in year t

$l_{i,t}$ = number of worker-equivalents of age i as of year t

L_t = number of people in the labor force

$M_{i,t}$ = number of males of age i as of year t

N_t = natural resources available for use in year t

P_t = total population in year t

R_t = the effective labor force; the sum of workers weighed by education

S_t = ratio of saving to output

w = ratio of children 20 and under to adults 21-60

Y_t = the aggregate output in year t

I. THE DEMOGRAPHIC STRUCTURES

The population and labor-force structures to be compared are as follows: The comparison base, structure I, has an exogenous 1% growth in the birth-rate each year, i.e., $F_{1,t}^I = 1.01F_{1,t-1}^I$, and $M_{1,t}^I = 1.01M_{1,t-1}^I$ starting in year $t = -60$. In this and in all other population structures infants live until they enter the labor force at age 21, and also through the end of their labor-force service at age 60, i.e., $M_{1,t}^I = M_{21,t+21}^I = M_{60,t+60}^I$, and the same for females. The number of males and females of each age are equal in this and in all other structures. (All children are assumed born on January 1, and up until the end of their first year the cohort is labeled (1_1)). Adults are assumed not to matter economically after age 60.

The population in year $t=0$ in structure I is then

$$\begin{aligned}
 (1) \quad P_{t=0}^I &= M_{60,t=0}^I + F_{60,t=0}^I + M_{59,t=0}^I + F_{59,t=0}^I \cdots + M_{1,t=0}^I + F_{1,t=0}^I \\
 &= M_{60,t=0}^I + F_{60,t=0}^I + 1.01 M_{60,t=0}^I + 1.01 M_{60,t=0}^I \\
 &\quad + (1.01)^2 M_{60,t=0}^I + (1.01)^2 F_{60,t=0}^I + (1.01)^{59} M_{60,t=0}^I \\
 &\quad + (1.01)^{59} F_{60,t=0}^I.
 \end{aligned}$$

In structure I, in which births increase 1% per year, half the women are assumed to work. The labor force at time $t=0$ is then

$$(2) \quad L_{t=0}^I = \sum_{i=21}^{60} M_{1,t=0}^I + .5 \sum_{i=21}^{60} F_{k,t=0}^I.$$

In structure II the population is augmented by a 50% increment in the birth rate in just a single year, $t=1$, i.e., $M_{1,t=1}^{II} = 1.51 M_{1,t=0}^{II} = 1.51 M_{1,t=0}^I$. All other cohorts remain the same as in structure I. Hence for the 40 years from $t=21$ to $t=60$ there is in structure II a single cohort that is roughly 50% larger than its next-aged cohorts, and the labor force is larger by that many workers for the 40 year period. This may be seen in Figure 1b, which shows the fine detail from Figure 1a for the first thirty years after $t=0$.

Figures 1a and 1b

In structure III the birth-rate is incremented by 50% over structure I in year $t=1$, but unlike structure II, the bulge continues in each successive year. That is, in structure III, $M_{1,t=1}^{III} = 1.51 M_{1,t=0}^I$,

and $M_{1,t=2}^{III} = 1.01 M_{1,t=1}^{III}$, and $M_{1,t=3}^{III} = 1.01 M_{1,t=2}^{III}$, and so on. Hence all cohorts from $t=+1$ onwards are more than 50% bigger in structure III than in structure I. The resulting labor force may be seen in Figures 1a and 1b. It is worth noting that after an adjustment period the dependency ratio, w , is again the same in structure III as in structure I.

In structure IV the birth-rate rises by 2% a year instead of the 1% in structure I, i.e., $M_{1,t+k}^{IV} = 1.02 M_{1,t+k-1}^{IV}$.

II. THE CAPITAL AND LABOR-FORCE EFFECTS OF HIGHER FERTILITY

Assume that the output of a large developed area such as the United States or all of the industrialized countries together is a Cobb-Douglas function such as

$$(3) \quad Y_t = A_k K_t^\alpha L_t^\beta.$$

The Effect Through the Supply of Parents' Labor

Incremental babies will cause some women to be out of the labor force who would otherwise work. From studies of U.S. census data by Bowen and Finegan (1969), Coin (1966), and Sweet (1970), together with the assumption that each woman will have at least one child, an incremental child is seen to result in a total decrease of .45 of a woman's work year, spread over the two years after the child is born. On the other hand, an incremental child causes a total increase in .10 of a man work year by fathers, spread over 25 years. The calculation for these estimates is given in Appendix A.

In the simulation runs in which the labor force is to be adjusted for the effect of children on the supply of labor,

$$\begin{aligned}
 (4) \quad L_t = & \sum_{i=21}^{60} M_{i,t} + .0025 (M_{1,t} + F_{1,t}) + .0025 (M_{2,t} + F_{2,t}) + \dots \\
 & \dots + .0025 (M_{25,t} + F_{25,t}) + .5 \sum_{i=21}^{60} F_{i,t} - .22 (M_{1,t} + F_{1,t}) \\
 & - .22 (M_{2,t} + F_{2,t}).
 \end{aligned}$$

The effect of incremental children on the parents' labor supply will be shown in the comparisons of structures II and III to structure I; in these cases all conditions are the same up to time $t=0$, and different thereafter as the numbers of births differ. But there seems to be no way to compare the labor-force effect of additional children in stable populations with different rates of growth, i.e., structure IV versus structure I.

The effect of incremental children on the parents' labor supply is not important, however, as can be seen in even an unrealistically-high upper-limit estimate of the effect of incremental children on the economy through the parents' labor-force. If the birth-rate is a low 25 per thousand and there are a low 400 employed workers per thousand, a doubling in the birth rate would only mean a drop in the labor-force to $(400 = .45 \times 25) = 389$, or about 3%, using an estimate of .45 worker-years lost per incremental child. Total output would drop even less, maybe 2%. Physical saving might then go down by, say $(.12 \times .02) = .0024$ of total output. The cumulative effect on output of such a change would be very small, and clearly it is thoroughly implausible that the birth

rate in an MDC might as much as double from any decadal base, say, to a plateau twice as high.

The Effect Through Changes in Private Saving of Physical Capital

Several kinds of evidence, discussed in Appendix B, are relevant for an estimate of the effect of number of children on private saving. These include family cross-sections, cross-sections of nations, and time-series evidence. One may find support for an estimate higher than -1.0, or as low as 0, for the elasticity of the proportion of income saved with respect to a proportional change in family size. Separate simulation runs were therefore made with elasticities of -1, -.5, and 0, as follows. The ratio,

$$w = \frac{\sum_{i=1}^{20} (F_{i,t} + M_{i,t})}{60 \sum_{i=21}^{60} (F_{i,t} + M_{i,t})},$$

is computed for each year in each case. For structure I it is .67 for each year, and is referred to as \hat{w} . In other structures the saving ratio for each year is then calculated as

(5) $S_t = \hat{S}(1 + e_c \frac{\hat{w} - w_t}{\hat{w}})$ where \hat{S} is the proportion of income saved in structure I, and e_s is the elasticity of saving with respect to children.

The Effects of Schooling

Two aspects of education are relevant. First, more children mean higher expenditures on education, which may cut into investment on physical capital as well as reducing consumption. Second, if incremental

expenditures on schooling are less than proportional to the number of incremental children and if there are no economies of scale in education, an increased number of children will cause a lower average quality of the work force in future years.

Education is treated in several ways. In the basic model, education is ignored completely, and investment in physical capital is 6% in all demographic structures.* In a second model, the level of education as measured by expenditures per child per year of school age is fixed and rising at 1% per year, because the annual increase in average school-leaving age has been of this general magnitude in the last half century. In the base year (and also for all other years in structure I) expenditure on education, $S_{t=0}^L$, is 6% (Appendix C gives the basis for this estimate). In all years

$$S_t = S_t^L + S_t^K.$$

In each year after $t=0$ the expenditure on education is made a function of the number of children

$$S_t^L = 1_{i=6,t} q_{i=6} + 1_{i=7,t} q_{i=7} \dots 1_{i=20,t} q_{i=20},$$

where the relationships among the expenditures for various school years, q_i , are fixed according to a crude schedule, e.g., grade 1 = 1, grade 2 = 1.125 ... grade 9 = 8 ... The effective labor represented by a worker

*The corresponding initial capital-output ratio is 3. Runs were also made with a savings rate of .12 and K/Y ratio of 4, with much the same results.

in any year is the square root of the total amount spent on his schooling during his youth.* And the effective labor force in any year, R_t , is the sum of the persons of labor-force age weighted by their effective labor values

$$R_t = \sum_{i=21}^{i=60} l_{i,t} \sum_{j=6}^{j=20} (q_{i,j})^{\frac{1}{2}}$$

where the subscript j refers to the various years in the past when the cohort received its education. Though the following equations show L rather than R , the latter should be understood for those models in which education is explicitly introduced.

In a third model, the level of education is not fixed exogenously. Rather, the total amount spent on education is made a function of the dependency ratio weighted by the relative school-year cost in each cohort

$$s_t^L = \frac{\sum_{i=6}^{i=20} l_{i,t}^{II} q_i}{\sum_{i=21}^{i=60} l_{i,t}^{II}} \bigg/ \frac{\sum_{i=6}^{i=20} l_{i,t}^I q_i}{\sum_{i=21}^{i=60} l_{i,t}^I}$$

where the Roman superscripts refer to demographic structures. This model suggests that the standard of education falls if the number of children rises. Again, effective labor-force, R , is entered into the production function in place of L where called for by the specific model.

*See Dennison (1969) for relationships between years of schooling and earnings, the latter a proxy for individual productivity. I do not think that Gintis' recent work (1971) contradicts this relationship over time.

A more refined model would change the proportions over time of each cohort getting education and working. But such a modification would not be likely to affect the particular sorts of conclusions this paper is intended to provide.

The effect on saving of the social spending for education and other children's services is most unclear. To my knowledge, there is no basis on which to estimate either the elasticity of spending on schools, or even harder, the extent to which the incremental expenditures on schools substitute for other social investment rather than causing new tax levies. I shall therefore simply assume that the three private saving elasticities being tried will bracket the elasticity that includes social as well as individual saving.

The Effect Through the Increments to the Labor Force

Now let us move ahead to the time when the incremental children enter the work force. If the capital stock does not receive an increment proportionally as large as the increment to the work force -- or, a fortiori, if the capital stock is even smaller than otherwise due to a reduction in saving -- then per-worker output will be lower than otherwise.*

*If the family and society save enough extra so that average capital per worker would be the same with or without the increment of children, as may be the case with the Hutterites, per-worker income would be the same after the incremental workers entered the work force. But this must occur at a cost of lower per consumer consumption prior to the years of labor force entrance.

The difference in early years between Y_t , the "base output," and Y'_t , the output with the incremental people in the work force, may be calculated as follows:

$$(6) \quad Y - Y' = AK^\alpha L^\beta - AK^\alpha L^\beta + \frac{Y}{L} \Delta L, = \beta \frac{Y}{L} \Delta L,$$

from the rule for the Cobb-Douglas function that the marginal productivity of labor equals average productivity multiplied by the coefficient of labor (Klein, 1962, p. 94). The difference in per-capita output is then easy to figure. In the longer run the "classical" effect of the incremental children is given by growth theory. Structures II and III, which have the same rate of fertility as structure I except for the initial shocks, converge in per-worker income to structure I after a period of lower Y/L . Structure IV must have a lower equilibrium rate of growth in per-worker income because of the higher fertility rate. The above propositions refer to the model in which the effective labor force is a function of the number of persons; when education also affects the effective labor force, things get more complex. And of course these propositions apply only when the rate of technological change is independent of the size of the population; this is not so in the models developed here.

III. KNOWLEDGE, SCALE, AND NATURAL RESOURCES

The previous section sketched the basic classical approach, which suggests that people other than the incremental population would be better off in at least the first 20 or 25 years if the incremental children were not born. Now we take up the forces that may mitigate or reverse

the effect: natural resources, economies of scale, and technical knowledge -- or NET for a short-cut acronym.

To disentangle the three NET factors from each other seems hopeless. Rather they must be treated together as a complex, and to doing so is a main methodological feature of this paper. To illustrate why they must be treated together, consider natural resources first. Natural resources might be thought of as a third factor of production.

$$(7) \quad Y_t = A K^\alpha L^\beta N_t^\gamma$$

and it would seem reasonable that N_t is a negative function of output in previous years, perhaps the sum of previous output

$$(8) \quad N_t = -g\left(\sum_{t=0}^t Y_t\right).$$

Equation (8) is consistent with the static physical point of view that natural resources such as coal and oil must diminish over time. But the definition of resources by the amount that are "really" in the earth is not operational and hence meaningless. What is relevant is that the economically meaningful available resources have mostly not decreased over time, as Barnett and Morse (1963) have shown. This increase in available resources is a function of increasing knowledge, e.g., new ways to prospect for and retrieve oil, new plastic materials to substitute for metals, and improved forestry techniques. Seen this way, natural resources are not different from physical capital. We may therefore think about the stock of available resources at time t as part of the capital factor, K , in equation (3), and the future course of the stock

of natural resources will be affected by saving and by increase in knowledge in the same way as conventionally-defined physical capital.

Considering economies of scale and technological knowledge, now: The two factors conceptually could be separated. One can imagine an experiment in which every other person and installation in the United States would be removed, holding the stock of knowledge constant, to see the effect upon output per worker. But such an experiment is not feasible, and the growth of scale and of knowledge have been so collinear in the past that it is not possible to separate them statistically. For this reason, and because of their essential inextricability,* we must treat them together.**

For purposes of estimation, one may identify the NET complex with the residual left in production-function studies after the effects of capital, labor and the amount of education are accounted for. In the context of the Cobb-Douglas production function, the residual may be seen as changes in A. The problem about whether the increases in capital (and labor) should or do reflect improvements due to increased

*Professor Kuznets emphasized this inextricability in conversation.

**Dennison's attempt (1967) to get at the effect of scale is useful but does not resolve this difficulty, I believe. In passing one might note that his estimate of the rate of advance of knowledge alone, is "much smaller than the increases in the population..., [which] implies a declining per capita contribution to knowledge" (1962, p. 237). In the context of this paper, it should be remembered that such advance in knowledge is only one of the sources of contribution to the NET complex.

knowledge is critical here, but we shall merely look the problem in the face and then pass rapidly on. One source for an estimate of the growth in per-worker output due to the increase in knowledge and scale, including the effect of natural resources, is Dennison (1967), who estimated the effect of elements roughly comparable to NET. For the period 1950-1962 for the U.S., Dennison estimated yearly growth of .76% for "advances in knowledge" (which excludes the effect of education on the labor force), and .30% for "economies of scale" (1967, p. 298), for a total just over 1%. For Northwest Europe he estimated .76% for "advances in knowledge," .56% for "changes in the lag in the application of knowledge, general efficiency, and errors and omissions," and .41% for economies of scale" (pp. 287 and 300), for a total of something over 1.5%. Solow's estimate for the U.S. for the 40 years from 1909 to 1949 is about 1.5% per year (1957, p. 316). Solow also adduces, though "not really comparable," an estimate of .75% per year from 1869-1948 by Valavanis-Vail, and Schmookler's estimate for 1904-13 to 1929-1938 which (though including agriculture) was of similar size to Solow's estimate. The model will be run with estimates of .5%, 1.0%, 1.5%, and 2.0%, to bracket any likely values.

The NET element is introduced as follows. If an increment of workers increases the work force by $\frac{\Delta}{L}$, one would expect that the NET residual which would otherwise be .01 would henceforth be $(.01 + .01\Delta)$ each year. But an increase in the knowledge component of NET does not

result in an instantaneous increase in productivity; rather, the effect of much knowledge is substantially lagged. The extent of the lag in the application of knowledge is an important empirical question concerning which I know of no evidence, though it would seem that the length of the lag is decreasing. Let us suppose that the present mean of the lag distribution for the NET complex as a whole, for an average cross-section of workers, is 5 years. This means that we can date the first increment to the productivity residual at 5 years after the incremental workers enter the labor force, with an additional increment to productivity in each of the following 40 years until five years after they retire. In the context of the model, the increase in A from year to year is proportional to the size of the labor force. This mechanism is calibrated so that the labor force at time $t=-5$ in structure I produces an X increase in A_t in year $t=0$, where X is whichever of .005, .01, .015 or .02 is being tried in that run. That is

$$(9) \quad \frac{A_t - A_{t-1}}{A_{t-1}} = bL_{t-5}$$

where b is chosen so that $A_{t=1} = (1 + X)A_{t=0}$ in Case I. The adjustment constant b is then used in the other cases. The point to notice here is that the NET additions, knowledge and economies of scale, from an increment of workers are cumulative.

IV. THE SYSTEM AS A WHOLE: A NUMERICAL MODEL

Before embarking on discussion of the dynamic model, it may be useful to show some static partial computations to illustrate the main forces operating. Assume that in the year $t=1$, and only in that year, the cohort of workers aged 21 is larger than in base case I, and hence the work force as a whole is larger than it would otherwise have been. Assume also that $\beta = 2/3$ and the base yearly increment due to gains in knowledge is 1%. If one calculates separately the drop in per-worker product due to the NET effect as in equation 9, in the year $t=5$ -- the first year in which this cohort's NET contribution is felt -- the downward push from the former effect is 32 times the upward push from the latter effect. But in year $t=6$ the drop from the capital effect is only 16 times the rise from the NET effect, because the incremental workers have now contributed two NET increments. In the year $t=7$ the ratio is 32 to 3. In less than 32 years the two effects would be roughly equal, and product per worker would be about what it would have been if the incremental workers had not entered the work force. From then on, product per worker is higher than it would otherwise have been.

Now let us consider, instead of this partial calculation, a dynamic model composed of the relationships described earlier in the paper. To recapitulate, the equations and the parameter estimates are as follows. First the production function, equation 3, where $\beta = .67$ and $\alpha = .33$. Next the net investment function, for simplicity a proportion of each year's income, where $c = .06$ (.12 in some runs)

$$(10) \quad K_{t+1} - K_t = cY_t.$$

The model begins in each case with $L_{t=0} = 1$ and $K_{t=0} = 1$. $A_{t=0}$ is set at 1/3. Separate runs were made with the savings elasticity at -1.0, -.5, and 0, and both with and without the adjustment for the parents' labor-force effect.

The full results for many variations with the basic no-education model are shown in Table 1. Summarized selected results from no-education and education models are shown in Table 2. The rates of growth from period to period will not be shown for other than the basic model; these "absolute" results were quite unrealistic because they were run with the same Cobb-Douglas exponents and other parameters as were used in the basic model, and more realistic models would require that these parameters be different when education is handled differently. But the relative values among the demographic structures can be meaningful, and are shown as percentages of the base demographic structure I. Also, only the runs with the "conservative" (i.e., biased-downward) estimates of the NET effect will be shown.

TABLES 1 and 2

1. The most important outcome is that under every set of conditions, even including all the runs with a base level of ΔA as low as .005, structures III and IV with more rapid population growth came to have higher per-worker income than structure I before the end of the simulation in year 160. And in every run, structure IV, which reaches

Table 1

Output per Worker, Initial Capital-Output Ratio of 3 and Initial Physical Savings Rate of .06, Labor Force, Not Adjusted for Parental Effect

Run	$A_t=0$	e_c	Case	00	10	20	30	40	50	60	70	80	90	100
	1.005	1.00	1	.33	.36	.39	.43	.47	.52	.57	.62	.69	.76	.84
	1.005	1.00	2	.33	.37	.40	.45	.49	.55	.60	.67	.74	.83	.92
	1.005	1.00	3	.33	.37	.40	.41	.45	.49	.56	.65	.75	.87	1.01
1	1.005	1.00	4	.33	.35	.37	.40	.44	.49	.55	.63	.73	.86	1.02
	1.010	1.00	1	.33	.38	.44	.50	.57	.66	.76	.88	1.01	1.17	1.35
	1.010	1.00	2	.33	.39	.45	.52	.60	.70	.82	.95	1.10	1.28	1.49
	1.010	1.00	3	.33	.38	.44	.48	.55	.64	.77	.96	1.18	1.44	1.76
2	1.010	1.00	4	.33	.37	.42	.48	.56	.66	.79	.96	1.19	1.49	1.88
	1.015	1.00	1	.33	.40	.48	.57	.68	.81	.97	1.15	1.37	1.63	1.93
	1.015	1.00	2	.33	.41	.49	.59	.72	.87	1.04	1.26	1.50	1.79	2.12
	1.015	1.00	3	.33	.40	.48	.55	.66	.81	1.01	1.30	1.66	2.09	2.61
3	1.015	1.00	4	.33	.39	.46	.55	.67	.84	1.05	1.33	1.71	2.21	2.88
	1.020	1.00	1	.33	.42	.52	.64	.79	.97	1.19	1.45	1.76	2.12	2.56
	1.020	1.00	2	.33	.42	.54	.67	.84	1.04	1.29	1.58	1.93	2.34	2.83
	1.020	1.00	3	.33	.42	.52	.62	.77	.97	1.26	1.67	2.18	2.81	3.57
4	1.020	1.00	4	.33	.41	.50	.63	.80	1.02	1.33	1.74	2.28	3.02	4.02
	1.005	.50	1	.33	.36	.39	.43	.47	.52	.57	.62	.69	.76	.84
	1.005	.50	2	.33	.36	.40	.44	.48	.53	.59	.65	.72	.80	.88
	1.005	.50	3	.33	.36	.40	.41	.44	.49	.55	.63	.73	.85	.98
5	1.005	.50	4	.33	.35	.37	.40	.44	.49	.55	.63	.73	.85	1.02
	1.010	.50	1	.33	.38	.44	.50	.57	.66	.76	.88	1.01	1.17	1.35
	1.010	.50	2	.33	.38	.44	.51	.59	.68	.79	.92	1.06	1.23	1.42
	1.010	.50	3	.33	.38	.44	.48	.55	.64	.76	.94	1.15	1.40	1.69
6	1.010	.50	4	.33	.37	.42	.48	.56	.66	.79	.96	1.19	1.48	1.88
	1.015	.50	1	.33	.40	.48	.57	.68	.81	.97	1.15	1.37	1.63	1.93
	1.015	.50	2	.33	.40	.48	.58	.70	.84	1.01	1.21	1.44	1.71	2.03
	1.015	.50	3	.33	.40	.48	.55	.65	.80	.99	1.27	1.61	2.02	2.51
7	1.015	.50	4	.33	.39	.46	.55	.67	.84	1.05	1.33	1.71	2.21	2.88
	1.020	.50	1	.33	.42	.52	.64	.79	.97	1.19	1.45	1.76	2.12	2.56
	1.020	.50	2	.33	.42	.53	.66	.82	1.01	1.24	1.52	1.85	2.24	2.70
	1.020	.50	3	.33	.42	.52	.62	.76	.96	1.24	1.83	2.11	2.71	3.43
8	1.020	.50	4	.33	.41	.50	.63	.80	1.02	1.33	1.73	2.28	3.02	4.01
	1.005	0.00	1	.33	.36	.39	.43	.47	.52	.57	.62	.69	.76	.84
	1.005	0.00	2	.33	.36	.39	.43	.47	.52	.57	.63	.69	.76	.85
	1.005	0.00	3	.33	.36	.39	.41	.44	.48	.54	.62	.71	.82	.94
9	1.005	0.00	4	.33	.35	.37	.40	.44	.49	.55	.63	.73	.85	1.02
	1.010	0.00	1	.33	.38	.44	.50	.57	.66	.76	.88	1.01	1.17	1.35
	1.010	0.00	2	.33	.38	.44	.50	.57	.66	.76	.88	1.02	1.18	1.36
	1.010	0.00	3	.33	.38	.44	.48	.55	.63	.75	.91	1.11	1.35	1.63
10	1.010	0.00	4	.33	.37	.42	.48	.56	.66	.79	.96	1.19	1.48	1.87
	1.015	0.00	1	.33	.40	.48	.57	.68	.81	.97	1.15	1.37	1.63	1.93
	1.015	0.00	2	.33	.40	.48	.57	.68	.82	.97	1.16	1.38	1.64	1.94
	1.015	0.00	3	.33	.40	.48	.55	.65	.79	.97	1.24	1.56	1.94	2.41
11	1.015	0.00	4	.33	.39	.46	.55	.67	.83	1.05	1.33	1.71	2.21	2.88
	1.020	0.00	1	.33	.42	.52	.64	.79	.97	1.19	1.45	1.76	2.12	2.56
	1.020	0.00	2	.33	.42	.52	.64	.79	.98	1.19	1.46	1.77	2.14	2.57
	1.020	0.00	3	.33	.42	.52	.62	.76	.95	1.21	1.58	2.04	2.60	3.28
12	1.020	0.00	4	.33	.41	.50	.63	.80	1.02	1.32	1.73	2.28	3.01	4.00

Table 1
(continued)

Run	$A_{t=0}$	e_c	Case	110	120	130	140	150	160	170
1	1.005	1.00	1	.94	1.05	1.17	1.31	1.47	1.65	1.86
	1.005	1.00	2	1.03	1.15	1.29	1.44	1.62	1.83	2.07
	1.005	1.00	3	1.17	1.36	1.58	1.82	2.11	2.45	2.83
	1.005	1.00	4	1.23	1.51	1.86	2.33	2.95	3.77	4.85
2	1.010	1.00	1	1.57	1.81	2.10	2.43	2.82	3.27	3.79
	1.010	1.00	2	1.72	2.00	2.32	2.69	3.13	3.63	4.21
	1.010	1.00	3	2.13	2.56	3.08	3.69	4.40	5.24	6.22
	1.010	1.00	4	2.40	3.08	3.99	5.21	6.82	8.99	11.88
3	1.015	1.00	1	2.28	2.69	3.18	3.75	4.41	5.19	6.10
	1.015	1.00	2	2.52	2.98	3.53	4.17	4.91	5.78	6.80
	1.015	1.00	3	3.24	3.99	4.87	5.93	7.17	8.64	10.37
	1.015	1.00	4	3.78	4.98	6.59	8.74	11.63	15.51	20.72
4	1.020	1.00	1	3.07	3.68	4.39	5.23	6.22	7.38	8.74
	1.020	1.00	2	3.40	4.08	4.89	5.83	6.94	8.23	9.76
	1.020	1.00	3	4.49	5.59	6.91	8.49	10.35	12.56	15.17
	1.020	1.00	4	5.35	7.15	9.57	12.83	17.21	23.11	31.05
5	1.005	.50	1	.94	1.05	1.17	1.31	1.47	1.65	1.86
	1.005	.50	2	.99	1.10	1.23	1.38	1.55	1.75	1.97
	1.005	.50	3	1.13	1.31	1.51	1.75	2.02	2.34	2.70
	1.005	.50	4	1.23	1.50	1.86	2.33	2.95	3.76	4.84
6	1.010	.50	1	1.57	1.81	2.10	2.43	2.82	3.27	3.79
	1.010	.50	2	1.65	1.91	2.22	2.57	2.98	3.46	4.01
	1.010	.50	3	2.04	2.46	2.95	3.52	4.20	4.99	5.92
	1.010	.50	4	2.39	3.08	3.99	5.20	6.81	8.97	11.86
7	1.015	.50	1	2.28	2.69	3.18	3.75	4.41	5.19	6.10
	1.015	.50	2	2.41	2.85	3.36	3.97	4.68	5.50	6.46
	1.015	.50	3	3.11	3.81	4.66	5.65	6.84	8.23	9.87
	1.015	.50	4	3.77	4.97	6.58	8.73	11.81	15.48	20.68
8	1.020	.50	1	3.07	3.68	4.39	5.23	6.22	7.38	8.74
	1.020	.50	2	3.25	3.89	4.65	5.55	6.60	7.83	9.27
	1.020	.50	3	4.30	5.35	6.60	8.09	9.86	11.96	14.43
	1.020	.50	4	5.34	7.14	9.56	12.81	17.19	23.08	31.00
9	1.005	0.00	1	.94	1.05	1.17	1.31	1.47	1.65	1.86
	1.005	0.00	2	.94	1.05	1.17	1.31	1.47	1.66	1.87
	1.005	0.00	3	1.09	1.25	1.45	1.67	1.92	2.22	2.57
	1.005	0.00	4	1.23	1.50	1.86	2.33	2.94	3.76	4.83
10	1.010	0.00	1	1.57	1.81	2.10	2.43	2.82	3.27	3.79
	1.010	0.00	2	1.57	1.82	2.11	2.44	2.83	3.27	3.80
	1.010	0.00	3	1.96	2.35	2.81	3.35	3.99	4.74	5.61
	1.010	0.00	4	2.39	3.07	3.98	5.19	6.80	8.96	11.84
11	1.015	0.00	1	2.28	2.69	3.18	3.75	4.41	5.19	6.10
	1.015	0.00	2	2.29	2.71	3.19	3.76	4.43	5.21	6.11
	1.015	0.00	3	2.97	3.63	4.43	5.37	6.48	7.80	9.34
	1.015	0.00	4	3.77	4.96	6.57	8.71	11.59	15.46	20.65
12	1.020	0.00	1	3.07	3.68	4.39	5.23	6.22	7.38	8.74
	1.020	0.00	2	3.09	3.70	4.41	5.25	6.24	7.41	8.77
	1.020	0.00	3	4.10	5.09	6.27	7.67	9.34	11.32	13.65
	1.020	0.00	4	5.34	7.13	9.54	12.79	17.16	23.04	30.95

NOTE: It is only in this model, and only in the high-savings-elasticity runs of this model, that Structure II differs from Structure I as much as it does. In the model that is identical except for $K/Y = 4$ and $S = .12$, this effect did not occur. The explanation is a puzzle, but an unimportant one, and does not seem to suggest any systematic error.

Table 2

Summary of Results of Selected Models

In all runs, initial $\frac{Y}{K} = 3$, initial $S_{t=1}^K = (K_{t+1} - K_{t=0}) = .06Y_t$, and elasticity of saving (e_c) = .50. The results shown are per-worker incomes in demographic structures II-IV as a proportion of $\frac{Y}{L}$ in structure I in the same year.

		t=0	t=20	t=40	t=80	t=160	Year in which crossing takes place
Basic NET feedback model, no allowance for education initial $\Delta A = 1.005A$	II	1.0	1.0	1.0	1.04	1.06	
	III	1.0	1.0	.93	1.06	1.42	60-70
	IV	1.0	.95	.93	1.06	2.28	60-70
Same except initial $\Delta A = 1.01A$	II	1.0	1.0	1.04	1.05	1.06	
	III	1.0	1.0	.88	1.14	1.37	60
	IV	1.0	.96	.98	1.18	2.74	50
No NET feedback, no allowance for education, $\Delta A = 1.01$ throughout	II	1.0	1.0	1.0	1.0	1.0	
	III	1.0	.97	.92	.92	.98	
	IV	1.0	.84	.87	.82	.78	
NET feedback, level of education fixed exogenously and expenditures on education a function of number of children, $S_t = S_t^K + S_t^L$ $Y = f(R)$, $A = f(L)$, initial $\Delta A = 1.005A$	II	1.0	1.0	.98	1.01	1.0	
	III	1.0	1.0	.91	1.03	1.35	70-80
	IV	1.0	.93	.89	1.0	2.17	80
Same except initial $\Delta A = 1.01A$	II	1.0	1.0	1.0	1.01	1.00	
	III	1.0	1.0	.90	1.09	1.46	60-70
	IV	1.0	.97	.90	1.11	2.61	60-70
Same except $Y = f(L)$, $A = f(R)$, initial $\Delta A = 1.01A$	II	1.0	1.0	1.0	1.0	1.0	
	III	1.0	.98	.92	1.04	1.44	70-80
	IV	1.0	.93	.92	1.03	2.62	70-80
Same except $Y = f(R)$, $A = f(R)$ initial $\Delta A = 1.01$	II	1.0	1.0	1.0	1.01	1.0	
	III	1.0	.98	.93	1.12	1.53	60-70
	IV	1.0	.94	.93	1.16	3.18	50-60
NET feedback, level of education an inverse function of dependency ratio weighted by relative school-year cost $Y = f(R)$, $A = f(L)$, initial $\Delta A = 1.005A$	II	1.0	1.0	1.0	1.01	1.01	
	III	1.0	.96	.80	.90	1.52	90-100
	IV	1.0	.89	.82	.84	2.39	100-110
Same as above except initial $\Delta A = 1.01A$	II	1.0	1.0	.99	1.01	1.01	
	III	1.0	.96	.83	.97	1.78	80-90
	IV	1.0	.90	.85	.95	3.42	80-90
Same as above except $Y = f(L)$, $A = f(R)$, $\Delta A = 1.01A$	II	1.0	1.0	1.0	1.01	1.00	
	III	1.0	.98	.87	1.03	1.74	70-80
	IV	1.0	1.04	.89	1.03	4.16	70-80
Same as above except $Y = f(R)$, $A = f(R)$, $\Delta A = 1.01A$	II	1.0	.98	.99	1.01	1.01	
	III	1.0	.94	.80	.95	2.92	80-90
	IV	1.0	.90	.85	1.03	27.9	70-80

a labor force (in millions, say) of 23,769 in year $t=160$, has a higher per-worker income than structure III, which reaches a total labor force of 7,346 in $t=160$. (For comparison, the labor force for structure I in year 160 is 4,913. In all structures the labor force is 1000 in year $t=0$.) These results may be compared with the classical growth-theory results seen in the third block in Table 2.

The mainspring that produces higher per-worker income with higher population is, of course, the element that makes the rate of change in the productivity coefficient a function of the number of persons in the work force. One might argue that this function would in the foreseeable future be even less than .005, or negative. But there seems to be no warrant for this argument in conventional studies of growth of national production using the GNP concept.

The higher the base rate of productivity change, the greater must be the relative final advantage of the cases of faster population growth, and the sooner the high-population-growth cases overtake the base case. And one sees that in run 1 (Table 1) of the no-education model which has the base A equal to 1.005 and elasticity of savings of -1.0, structure III overtakes structure I between periods 60 and 70 and structure IV does the same. In otherwise-similar run 4 where the base A is 1.02, structure III overtakes structure I in period 50, and structure IV overtakes it between period 30 and 40.

2. The effect of incremental children on savings can have substantial impact on the results in structure IV. In the no-education

model with $S_t = .12$ and $K/Y = 4$ (results not shown) by year 160, the comparison of the -1.0 savings elasticity with the zero elasticity shows such ratios as 3.65/4.42, 8.94/10.94, 15.59/18.93, and 23.36/28.38, all a bit over 4/5. The -0.5 elasticity produced results roughly in between the zero elasticity and the -1.0 elasticity. For structure III, however, the savings effect is very small relative to the differences in Y/L between structure III and structure I. On the other hand, the effect is less when $S_t = .06$, as may be expected.

3. As seen in the comparison of structure III with structure I in the runs with and without the labor-force adjustment, the effect through the parents' labor supply of incremental children after the first child is quite insignificant, just as preliminary calculations had suggested they would be. In no case was the relationship between structures I and III as much as a quarter of a percent different in year 160 between the runs that were and were not adjusted for the parents'-labor-force effect. Hence only the runs without the parents'-labor-force effect are shown.

4. The time required for Y/L in structures III and IV to overtake Y/L in structure I is generally longer in models where expenditures on education affect physical saving, even where education positively influences both R_t and A_t . But this is not invariably true, especially for structure IV where the labor force always has a younger average age and hence may have a higher average education than in structure I, because of the secular growth in education.

V. SUMMARY AND DISCUSSION

1. Increases in productivity as a result of increased scale and of knowledge caused by increases in population were added to a simple classical model of an MDC. Under assumptions about the parameters that I hope are reasonable, demographic structures with larger rates of population growth, after initially falling behind in per capita income, usually overtake structures with lower rates of population growth in much less than a century, less than half a century in some cases. This outcome is a step toward quantifying Kuznets' reasoning about the role of knowledge in modern economic growth.

2. No distinction has been made between market-induced and market-autonomous productivity increases. One reason is that the variation explained by economic incentives that induce innovation is much greater within a given industry than within a society as a whole; the reward structure has more influence on whether an inventor works on airplanes instead of railroads than it does on whether he innovates or does not innovate at all, it would seem. Another reason is that the incentives are endogenous, and hence are most easily treated as innards of the black box that is considered here only in its over-all shape and behavior.

3. Some persons will criticize this formulation of the NET effect on the grounds that the past rate of increase in knowledge, economies of scale, and productivity may not continue in the future. Perhaps. But, even if so, this formulation should add to our understanding

of the growth of population and per capita income in the past history of the U.S. and Western Europe. And for at least a short period in the future it does not seem unreasonable to project the long term trend of the past. Farther into the future we must bring other arguments to bear to help us decide whether the growth of productivity will be faster or slower than in the past. The same criticism may also be made about natural resources in the future, with the same response.

Some writers have argued that there are already diseconomies of scale operating, pointing out that traffic jams and other congestion phenomena increase at a faster rate than the number of people involved. Whatever the truth about the effects of scale in various separate aspects of the economy, it seems to me that the overall measure of the NET, the increase in productivity, is the most meaningful economic measure.

4. The physical capital-output ratio is falling over time in the U.S., due to the shift to tertiary activities and to the discovery of better ways to make capital equipment. But on the other hand, the social cost of schooling will rise in the future. So on balance one does not know whether the social cost of an incremental labor-force entrant will fall or rise in the future, relative to his earnings.

5. The difference in effects of population increase in less-developed and more-developed countries comes out sharply in this analysis. Productivity per worker does not grow much from year to year in many LDC's, and hence the residual is small. This implies that an increase in workers will not increase productivity per worker through an increase

in knowledge. This conclusion is made even stronger by the fact that a considerable portion of the increase in knowledge operative in LDC productivity occurs outside any LDC, and is rather independent of the size of the LDC work force.

6. The dependent variable in this work has been output per worker measured in conventional national-income terms. If such amenities as space and purity of the environment were included in the measurement, the results might be different. (Please note, however, that the effect of added population on some amenities, such as variety, may be positive.) Perhaps the most reasonable way to handle the problem would be to deduct for the hypothesized costs of maintaining a constant level of such amenities. But this is quite beyond scientific capability at the moment. There is no reason to believe, however, that such an addition to the reckoning would change the relative results of population growth, because there is no reason to believe that at any given point in time there was or will be a reversal or major change in the trend of the effect of added population on such matters as space and pollution.

Appendix A

ESTIMATE OF THE EFFECT OF INCREMENTAL CHILDREN ON LABOR SUPPLIED BY MOTHER AND FATHER

First let us consider the effect of incremental children on the work supplied outside the home by the mother. The basis is the body of work on the U.S. Census of 1960 by Bowen and Finegan (1969), Cain (1966), and Sweet (1970). The effect is greater in the years right after the children are born. By the time children are 12, there is no observed difference between the labor-force participation of the incremental mothers and of matching women who did not have the incremental births - due surely to a trade-off between the labor-increasing effect of a greater "need" for money, and a labor-reducing effect of the continued need for care by the child. There may also be a negative effect from decay of the woman's skills while she is out of the labor force for one more baby.

To obtain an order-of-magnitude estimate for use here, assume that each woman will have at least one child in any case, and that each incremental baby means that the mother will for two years more than otherwise have ~~a~~child under twelve. Hence for two years less than otherwise she will have no child under 12, the child's age at which labor-force participation ceases to be much affected by the presence of the child. This means for each incremental child two years of ^{mother's} labor force participation at, say, 15% rather than the same two years at say, 42% labor-force participation (Sweet, 1968, p. 99). About 2/3 of all women who work are full-time workers, and we shall assume part-time workers work half-time. Rough calculations then suggest that an incremental baby results in an over-all loss of $2(.42 - .15)(\frac{5}{6}) = .45$ years of work, or

.225 of a worker "lost" to the work force in each of the two years after an incremental baby is born.

Another approach is to figure that women with children under 6 work an average of 5.6 hours, whereas women with no children under 18 work an average of 15.5 hours (Sweet, 1968, p. 130). This suggests a loss of a total of 10 hours' work a week for two years, or again $2 \times \frac{10}{44} = .45$ years of work.

The over-all effect computed here may seem to be small. If this is so it is largely due to the assumption - reasonable, in my judgment, given the present incidence and trend of childlessness in the United States - that women will have at least one child. There is a very big difference in the propensity to work of women with no and some children, and much less difference among propensities to work of women with different positive parities. And even this estimate probably is biased upwards, because some women choose to have more children on account of an already-made decision not to work rather than vice versa. If so, it is wrong to interpret the observed difference in labor-force participation as completely caused by the number of children. Still another reason why the true net negative effect of children on women's work is probably less than the estimate of .45 work years per child is that women who work often employ substitutes to do their domestic chores.

Now we move from the mother to the father. The positive labor-force effect of additional children on men, and also perhaps on some groups of women whose children are 12 or over, receives less emphasis than the negative effect on women, perhaps because the linkage seems less mechanical and more psychological, being a shift in preferences concerning work. Yet Bowen and Finegan's work on labor force participation is shot through with examples of "need" increasing labor supply, e.g., the strong effect of husband's income on wife's propensity

to work. Another example of the effect of changed need is Clark's finding that the greater the war damage a country suffered in World War II, the higher its rate of saving after the war (1967, p. 268). The fuzziness of the phenomena that cause a positive effect here should not lead us to downgrade their importance.

From the 1/1000 1960 US Census tape I regressed men's hours worked per week on the number of children men have, holding constant with sub-classification and multiple regression the men's education, race, age, occupation and residence area. For white men an additional child is associated with approximately .2 additional hours of work per week. If one assumes a 44 hour week and 25 years of work after the incremental baby is born, then $\frac{.2}{44} \times 25 \approx .10$ additional work years result per child. This must be an understatement of the effect, however, because of the bias introduced by the error in the dependent variable. The error in the male-hours-worked estimate due to the variation from week to week in hours worked must be considerable, which would cause the regression coefficient to be biased downwards.

Other relevant evidence comes from moonlighting. According to Guthrie's summary of the literature (1966), during the years when men have young children - between the ages of 24-44 - the rate of moonlighting is relatively higher. And the incidence of moonlighting among a sample of army men is strongly affected by the number of children they have. Crude calculations on that data indicate that an extra child is associated with an increase of about 1/400 in the amount of moonlighting work done each year by the child's father, or about a tenth of an hour per week on the average. (It is also relevant that almost three times as many moonlight jobs than regular jobs are self-employment. Such enterprise must be good for any economy and society.)

Despite the above additions and qualifications, the model uses the "conservative" estimate derived from the 1960 Census data tape of .2 hours per week more male labor per additional child for 22 years after the child's birth, or a total of .10 additional work years per child.

To summarize, the loss in women's labor is a total of .45 years and the gain in men's labor is .10 years, for an additional child a difference of .35 man years. However, the losses through the mothers occur at an earlier time than do the gains through the fathers. Hence the net loss (if the estimates are right) is greater than the difference of .35 man-years suggests.

As seen in Equation 4, the parents'-labor-force effect is introduced as follows. By the reasoning given earlier, for each baby born, .225 of a worker is lost to the work-force for each of two years. Therefore, .225 of a worker is removed from the model's labor force in the first and second years of each incremental child's life.* Each father is estimated to offer .25% more work per incremental child, or an increase of .0025 of a worker, and hence that much work is added in each of 25 years following the incremental child's birth.

* The labor force in Case I, year $t=0$, is assumed to be composed of 66% men and 34% women.

Appendix B

ESTIMATES OF THE EFFECT OF INCREMENTAL CHILDREN ON FAMILY SAVING AS A PROPORTION OF INCOME

1. Family Budget Surveys.

Budget surveys over a cross-section of families are one source of evidence. Typically, families are classified by income; within each income bracket, family size is the independent variable and savings the dependent variable. Brady (1956, reviewed in Coale, 1960) examined six surveys over sixty years in the U.S. She concluded that the elasticity of consumption with respect to family size is $1/6$. If one assumes that the marginal propensity to consume is .88, her finding translates into an elasticity of -1.2 for saving.*

Eizenga allowed for the effect of age and income with a technique of "multiple standardization" before estimating the effects of family size on savings from family cross-section data. He estimated that in 1950 savings would have been about \$31 less if the family had 5 children rather than 4, and about \$50 less for the average family with 6, 7, 8 or more children than for the family with 4 children (1961, p. 90). Relative to per-family income in that year, these amounts do not seem large, from any point of view. These estimates may be biased downwards, however, because of the nature of the sample (the Consumer Expenditure Study of 1950).**

*That is, if a 6% increase in family size produces a 1% increase in consumption from .88 to .89, the reduction in saving is $1/12$ or about 8%, and elasticity is 8%/6%.

**Consumer surveys must also contain an upward bias because they customarily omit social security which - like pension-fund saving - is fixed outside the family independently of the family size.

There are several reasons why the relationship between family size and saving observed in budget surveys may not be causal or meaningful for purposes here:

a) There is a statistical reason to believe that the observed relationship between family size and saving is not causal. Consider now the relationship between income and family size. In heterogeneous groupings such as the US or another country as a whole, the relationship is inverse. And the relationship between current income and proportion saved is positive (e.g., Friedman, 1957). Therefore, the simple correlation between family size and proportion saved almost inevitably is negative.

But within a much more homogeneous group of people, e.g., a group defined by income, occupation, urbanity ~~of~~ residence, and age, the relationship - a much more "partial" one than in the heterogeneous case - between income and fertility is observed to be positive.* Would it not be reasonable, then, to expect the negative relationship between family size and saving to disappear, and perhaps a positive (but also non-causally interpretable) correlation also to appear in such a group? And it certainly seems reasonable that the appropriate context in which to think about the effect of children on family size is the homogeneous group - that is, with all else held equal - because it is rather unlikely that number of children is an important causal antecedent to changes in homogeneity of groups with respect to education, age, and so on.

b) If more children cause higher absolute income - as they may after the first few years after a birth, as the father's labor effect dominates - the proportion saved in the budget cross-section is biased upward.

* For the US see Ruggles and Ruggles, 1960; Simon, 1969; Willis, 1969.

c) Within cultural groups there may be something analogous to the "common set of factors residing in the political and social institutions of a country and in the views governing the behavior of its inhabitants [that] determines both the economic performance and its demographic patterns" (Kuznets, 1965, p. 29). To the extent that this is so, one should not interpret the observed relationship between family size and the savings ratio as a causal relationship.

d) Budget surveys are unlikely to capture the effect of the birth rate on business investment, either by family entrepreneurs or by incorporated businesses. The increase in total output due to the incremental future workers raises the expected return on investment, and hence brings some investment projections above where the cut-off would otherwise be.

2. Cross-National Comparisons

In a cross-section comparison of countries, Leff (1969) found the elasticity of saving with respect to dependent children to be $-.43$ in MDC's, when controlling for per capita income and other variables. Leff's method is unexceptionable in itself. But it is well to keep in mind the contradictory results obtained by the several international multivariate cross-section studies of a related matter, the connection of income and family size. Some studies have found a negative coefficient (Russett et al., 1964; Adelman and Morris, 1966; Rao and Dey, 1968), others a positive coefficient (Weintraub, 1962; Adelman, 1963; Heer, 1966; Friedlander and Silver, 1967). Hence any single study of this sort should be treated with caution.

3. Long-run Time-Series Evidence

The notion of an inverse savings-family size relationship receives no support from the observed sharp decrease in family size in the U.S. in

the last 100 or 50 years, without secular increase in the proportion of saving to income. But because so many other things also changed during that period of time, it would be most unwise to interpret these data as showing that fewer children produce less saving.

In historical perspective, too, one might expect the relationship of savings to family size to change as a country becomes more modern. New needs for saving arise and old substitutes for personal saving disappear. Contemporary middle-class families feel a strong need to save for the college educations of their children, and an increase in offspring might increase saving on this account; casual support is provided by the behavior of life insurance salesmen, who descend upon families after marriages and births. Furthermore, the additional children are not thought of as reducing the parents' need for retirement saving, as in pre-modern times. These are two strong reasons to expect the savings-family size relationship to be more positive in richer, more modern economies than in poorer situations.

4. Other Considerations

The aspect of savings behavior in connection with population which has caught the interest of most theorists (e.g., Cassell, 1932ⁱⁿ/Phelps, 1969; Kuznets, 1960; Meade, 1955; Modigliani, 1965; and Phelps, 1969) has been the life-cycle effect. If a population's earners are relatively young, on the average they will save more than an older population because saving takes place earlier in life than does dissaving. But one should not interpret this phenomenon as suggesting that an increase in fertility will increase saving, because this comparison omits the possible effect of the pre-labor-force years on saving behavior. At the level of aggregation used in growth theory it

would be extremely difficult to relate saving to the effect of an incremental birth. Furthermore, this sort of analysis deals in proportions rather than in total savings, which are of interest here. Hence I think this life-cycle aggregate analysis is not fruitful here.*

An increase in family size may actually increase total saving in some cases. An example: The Hutterites of North America who live communally in colonies "do not believe in practicing birth control, and so they continue to increase and thus to create the need for additional colony sites. The colonies need more and more cash in order to buy more and more land. This reduces the amount of cash available for other things...Young colonies starting out, or any colony unable to realize the levels of productivity needed for saving, will be helped by others" (Bennett, 1967, pp. 164-165).

In general the theory of consumption and savings is most complex and unsettled at present. One cannot say with much certainty what will be the effect of an increase in income of any sort (e.g., short-run, long-run, windfall) on saving and consumption. And in particular the relationship of family size to saving must be even more complex and unsettled than consumption theory in general.

*For the same reason, Clark's finding across a sample of countries of no relationship between population growth and the savings ratio, holding per capita income constant (1967, p. 268) is not helpful for our purposes here.

Appendix C

ESTIMATE OF THE EFFECT OF INCREMENTAL CHILDREN ON SOCIAL EXPENDITURE

Schooling is the only social expenditure considered here. The bases for the estimate of the effect of children on public schooling costs are as follows: (a) Expenditure on education is 4.6% of US national income (Harbison and Myers, 1969, p. 41); (b) A quarter of the population is in school, 18.4% of the population being in the 5-14 age group (ibid); (c) In 1968, \$623 per year was spent by public schools per student year (Statistical Abstract, 1969, p. 102); (d) \$6,856 average year-round male earnings in 1966 (Statistical Abstract, 1969, p. 233). This estimate excludes foregone earnings, on-the-job training costs, etc.; (e) A high-side inclusive estimate of US education plus training costs is 12.9% of adjusted GNP, by Machlup (Harbison and Myers, 1964, p. 28n).

How much of children's education expenditures should be considered as consumption is a matter not considered here.

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